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Optimization of Johnson–Cook material model constants for OFHC copper

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Abstract. Numerical modeling of processes of high-speed and shock-wave interaction of deformable solids is an urgent task. The adequacy and accuracy of numerical results for such problems depend on the accuracy of the material model, which relates the yield strength to the accumulated plastic strain, strain rate, and temperature. The most commonly used empirical model in engineering applications is the Johnson-Cook (JC) model. However, the model parameters are determined experimentally at low strain rates, which leads to a large deviation of calculations from experiment when modeling high-speed impact. At the same time, it is not possible to provide direct measurement of JC model parameters at high strain rates. This paper develops a method for estimating the constants of the JC model using the optimization algorithm by the Nesterov gradient method, based on the results of comparing numerical calculations with experimental Taylor tests. It is shown that the optimized JC model constants more accurately describe the material behavior under high-speed deformation.
Keywords: JC model, optimization, numerical modelling.

1. Introduction

The shock-wave interaction of deformable bodies is actively investigated to date [1–5]. The Johnson-Cook constitutive model (JC) [6–8] is a frequently used hardening model for numerical modeling of problems associated with high strain rates. This model connects yield strength with accumulated plastic strain, strain rate, and temperature. As the strain rate increases above 10^3 s^{-1}, there are increasing difficulties in choosing the constants of this model to ensure that the numerical and experimental data are in good agreement.

To test numerical models and to select constants of constitutive models, the Taylor test is often used [9]. This method is especially useful for experimental studies (impact of a cylinder on a non-deformable obstacle at relatively low throwing velocities) [1, 5, 10–12] and for numerical modeling in three dimensions [13–15]. In the last case, there is no need to calculate the contact surfaces of interacting bodies, which significantly simplifies the numerical modeling code.

In this paper, numerical calculations of the classical Taylor problem for cylindrical samples made of copper OFHC (analog M1) are carried out. A method of choosing the JC model constants using an optimization algorithm based on Nesterov's gradient descent method [16] is proposed. To optimize the process of choosing the JC model constants, the solution quality functional [13] is used, which can be used to quantitatively assess the degree of deviation of the calculation results from the experimental data and to determine the optimal parameters of the JC model. The optimal parameters of the JC model for the experimental data set were selected.

2. Mathematical model

2.1. Equations of motion

The system of equations consists of the equations of continuity (1), motion (2), and energy (3) [17, 18]:

$$\frac{\partial \rho}{\partial t} + \operatorname{div}(\rho \upsilon) = 0, \tag{1}$$

$$\rho \mathrm{d}\upsilon_i / \mathrm{d}t = \sigma_{ij,j}, \tag{2}$$

$$\frac{\mathrm{d}E}{\mathrm{d}t} = \frac{1}{\rho}\sigma_{ij}\varepsilon_{ij}, \tag{3}$$

where v_i – components of the velocity vector , v, ρ – density, E – specific internal energy, $\sigma_{ij} = -P\delta_{ij}+S_{ij}$ – components of the stress tensor, ε_{ij} - components of the strain rate tensor, S_{ij} – components of the stress deviator, P – pressure.

2.2. Equations of state

Stresses and strain rates are related by the relation

$$2G\left(\varepsilon_{ij}-\tfrac{1}{3}\varepsilon_{kk}\delta_{ij}\right)=\frac{\mathrm{d}S_{ij}^{0}}{\mathrm{d}t}+\lambda S_{ij}\,, \tag{4}$$

$$\frac{\mathrm{d}S_{ij}^{0}}{\mathrm{d}t}=\frac{\mathrm{d}S_{ij}}{\mathrm{d}t}-S_{jk}\omega_{ik}-S_{ik}\omega_{jk}\,, \tag{5}$$

where G – shear modulus, δ_{ij} – Kronecker symbol, dS^0_{ij}/dt – derivative of Jaumann. Parameter λ at plastic deformation is determined using the Mises yield condition

$$S_{ij}S_{ij}=\frac{2}{3}\sigma^{2}, \tag{6}$$

yield strength σ is determined using the Johnson-Cook model [6]

$$\sigma'=\left(\sigma_0+Be_{eq}^{n}\right)\left(1+C\ln\frac{\varepsilon_{eq}}{\varepsilon_0}\right)\left(1-\left(\frac{T-T_0}{T_m-T_0}\right)^{m}\right), \tag{7}$$

where T_0, T, T_m – initial, current temperature and melting temperature, respectively, e_{eq} – equivalent strain, ε_0 – initial value of equivalent strain rate (ε_0=1 c^{-1}), ε_{eq} – current equivalent strain rate, B, C, n, m – model constants.

Temperature and specific heat capacity were calculated using relations [10]:

$$dT=\begin{cases} dE_{sh}/c_p\,, & if\quad T<T_m \\ 0\,, & if\quad T=T_m \\ d(E_{sh}-\Delta H_m)/c_p, & if\quad T>T_m \end{cases}, \tag{8}$$

$$c_p=\begin{cases} c_p^0+\left(c_p^L-c_p^0\right)\dfrac{T-T_0}{T_m-T_0}, & if\quad T_0\le T<T_m \\ c_p^L\,, & if\quad T\ge T_m \end{cases}, \tag{9}$$

where E_{sh} – shear energy, E_{0x} – cold component of specific internal energy, E – specific internal energy, ΔH_m – specific heat of melting, $c_p{}^0$ – specific heat capacity in normal condition, $c_p{}^L$ – specific heat capacity in melted state.

The pressure P_c in an intact material is determined using the Mie-Grüneisen type equation of state:

$$\begin{aligned} P_c &= \rho_0 a^2\mu+\rho_0 a^2[1-\gamma_0/2+2(b-1)]\,\mu^2+ \\ &+\rho_0 a^2[2(1-\gamma_0/2)(b-1)+3(b-1)^2]\,\mu^3+\gamma_0\rho_0 E, \end{aligned}, \tag{10}$$

where a and b are the Hugoniot adiabatic constants $u_s = a + bu_p$, ρ_0 is the initial density of the material, γ_0 is the Grüneisen coefficient, $\mu = (V_0/V)-1$, V_0 and V are the initial and current specific volumes, u_s is the shock velocity, u_p is the mass velocity of the media behind the shock front.

3. Experiments and calculations

The interaction of a cylindrical impactor with a non-deformable barrier (Taylor test) is numerically modeled in a three-dimensional formulation. The material of the cylindrical specimen (copper OFHC (M1)) is chosen to compare the numerical results with the experimental data [5], the parameters of the experiments are given in Table 1. The impact velocities investigated were 162, 167, 225 and 316 m/s. For numerical calculations within the elastic-plastic model of the medium, our own research software COMP3 based on the modified finite element method [17] was used.

Table 1. Parameters of experiments.

№	Material	L_0, mm	D_0, mm	v_0, m/s	source
1	OFHC Cu cylinder	34.5	7.8	162	[5]
2				167	
3				225	
4				316	

Table 2. Initial parameters of the Johnson-Cook model [6].

σ_0 (MPa)	B (MPa)	C	n	m
89	292	0.025	0.31	1.09

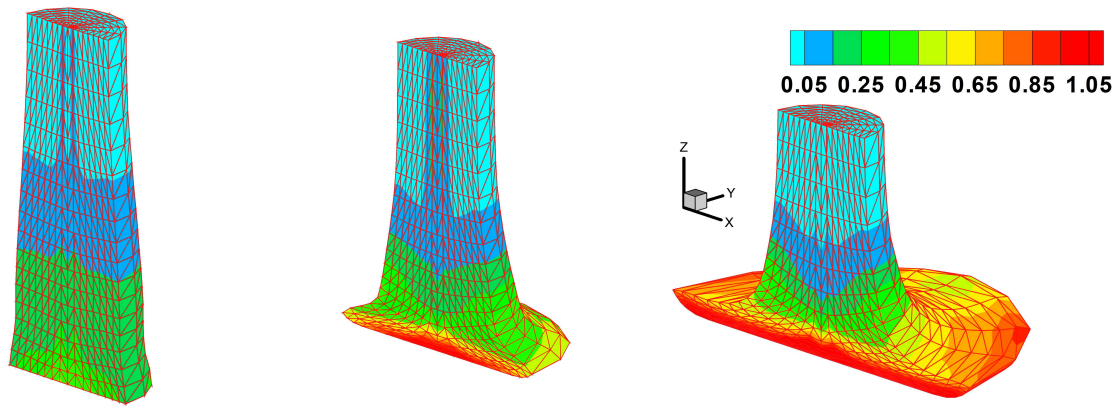
To measure the deviation between experimental and numerical results, the solution quality functional Q_f, proposed in [13], is used. The task of choosing the parameters of the JC model is to find the minimum of Q_f when the parameters B and c vary.

Optimization of JC model parameters was performed for the following problems (Table 1):

1) For each experiment 1–4 individually;

2) For experiments 1–4 in combination.

Fig. 1 shows the calculation grid and shear strain specific energy fields for the three initial impact velocities. The end of calculations in each case occurred at the moment of time when the average velocity of the specimen along the Z axis became positive. The maximum shear strains are formed at the specimen – rigid wall contact boundary in the center of the cylindrical specimen. For an impact velocity of 316 m/s (Fig. 1c), intense shear deformations cover the entire contact layer of the cylinder.



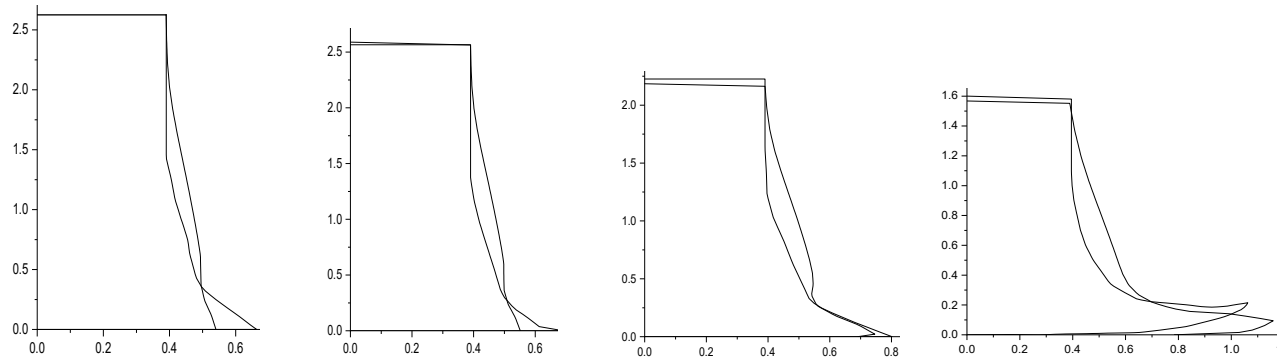
(a) (b) (c)

Fig. 1. Calculation grid and specific energy fields of shear strain (in GJ/m^3) in a cylindrical specimen during impact on a rigid wall: a – initial velocity 167 m/s, time moment 78 μs; b – initial velocity 225 m/s, time moment 85 μs; c – initial velocity 316 m/s, time moment 87 μs.

A procedure to find optimized parameters of the JC model [13] was carried out for the specified experimental data. The results of comparison of calculations with experiments for the initial model parameters and optimized ones are shown in Fig. 2 and 3.

Figs. 2 and 3 show the experimental and calculated cylinder contours for four impact velocities. In Fig. 2, the simulation results are shown for the case of original JC model constants, while in Fig. 3 – for the optimized values of constants *B*, c of the modified JC model.

The parameters of the JC model obtained as a result of optimization are presented in Table 3.



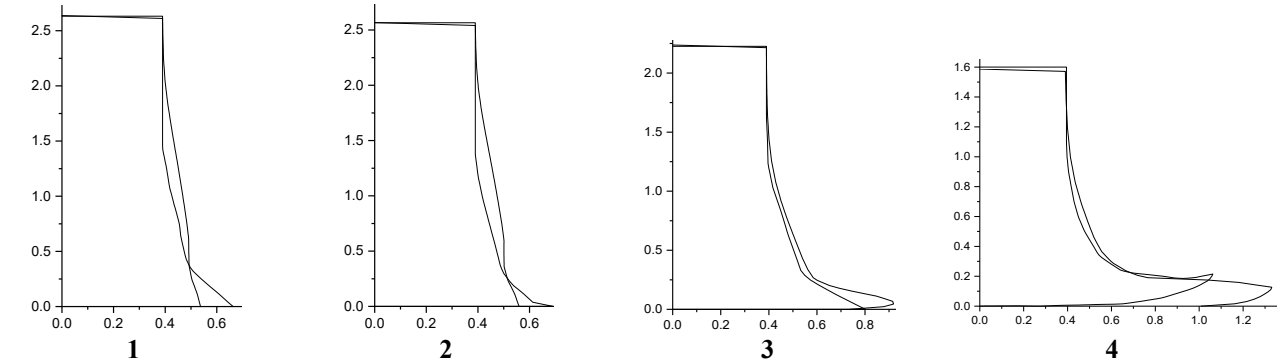
1 **2** **3** **4**

Fig. 2. Calculated and experimental profiles of back and side surfaces of cylinders for tests 1–4, parameters of the classical JC model are taken from Table 2.

Optimal parameters of the investigated copper are given in Table 3.

Table 3. Optimal parameters of the modified JC model.

Experiment	***B* (GPa)**	***C***
1	0.309	0.024
2	0.286	0.022
3	0.539	0.014
4	0.486	0.018
1+2+3+4	0.565	0.020



1 **2** **3** **4**

Fig. 3. Calculated and experimental back and side profiles for tests 1-4, parameters of the modified JC model are taken from Table 3.

A comparison of the deviation of the calculated cylinder surface profiles after impact is shown in Table 4.

Table 4: Quality of solution.

Experiment	Q_f	Q_{f0}	Q_{f0}/Q_f
1	0.098	0.099	1.0
2	0.143	0.143	1.0
3	0.041	0.162	3.9
4	0.088	0.311	3.5
1+2+3+4	0.063	0.179	2.8

4. Conclusions

The original JC model parameters describe well the behavior of OFHC copper (M1) in the experiments [5] at impact velocities of 162 and 167 m/s, but at higher impact velocities the deviation of the optimal model parameters from those determined by low-velocity experiments increases up to 2 times.

The determination of a set of parameters optimally describing the whole set of the considered experiments showed that the solution quality improves by a factor of 2.8 in terms of the used solution quality functional.

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